
Summary

We investigate statistical properties of several classes of periodic billiard models which can be regarded as diffusive. We begin by motivating the study of such models in Chap. 1 and reviewing how statistical properties arise in Chap. 2.

In Chap. 3 we consider a periodic Lorentz gas satisfying a geometrical condition, for which diffusion has been rigorously proved. We discuss how to estimate diffusion coefficients from numerical data and then study their geometry dependence, finding a qualitative change in the shape of curves as one parameter is varied. We discuss the application of a random walk approximation of the diffusion coefficient and a related Green–Kubo formula. We also consider the effect on the diffusion coefficient of reducing the geometrical symmetry.

In Chap. 4 we study the shape of position and displacement distributions, which converge to a normal distribution by the central limit theorem. We find a fine-scale oscillation in the densities which prevents them from converging pointwise to Gaussian densities, and relate this to the geometry of the billiard domain, giving an analytical expression for the fine-structure function. We provide strong evidence that, when demodulated, the densities converge uniformly to Gaussians, strengthening the standard central limit theorem, and we find an upper bound on the rate of this convergence. We further consider the effect of a non-constant distribution of particle speeds, showing that the limiting position distributions can be non-Gaussian.

Chap. 5 treats polygonal billiard channels, where few rigorous results are known. We provide numerical evidence that normal diffusion can occur, and that the central limit theorem can be satisfied. We develop a picture of how normal diffusion can fail if there are parallel scatterers, and we characterise the resulting anomalous diffusion, as well as the crossover from normal to anomalous diffusion as such a geometrical configuration is approached.

In Chap. 6 we extend our methods to a three-dimensional periodic Lorentz gas. We present a model with overlapping scatterers exhibiting normal diffusion in a certain regime. Outside this regime we provide evidence that the type of holes present in the structure strongly influences the statistical properties, and show that normal diffusion may be a possibility even in the presence of cylindrical holes.

We finish in Chap. 7 with conclusions and some directions for future research.