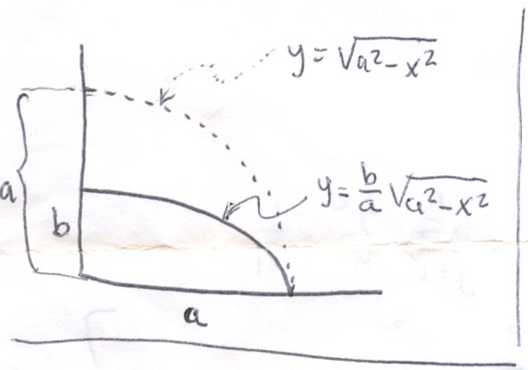
Sean $a, b \in \mathbb{R}$ t.q. $0 < b < a$. Consideremos la elipse $\mathcal{E}$ dada por $\{(x,y) \in \mathbb{R}^2 \mid \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\}$; (con semiejes a, b)

Sean C la circunferencia dada por $x^2 + y^2 = a^2$, $R = \{(x,y) \mid x \geq 0,$
Por simetría tenemos $\text{Área}(C) = 4\, \text{Área}(C \cap R)$ y similarmente $\text{Área}(\mathcal{E}) = 4\, \text{área}(\mathcal{E} \cap R)$. Sean $A_1 = \text{Área}(C \cap R)$, $A_2 = \text{Área}(\mathcal{E} \cap$

Sabemos que $4A_1 = \pi a^2$. Para obtener A_2, consideremos la partición regular $P_n = \{0, \frac{a}{n}, \frac{2a}{n}, \ldots, \frac{(n-1)a}{n}, a\}$, tenemos

$A_2 = \lim_{n \to \infty} L(f, P_n)$, donde $f : [0,a] \to \mathbb{R}$ es $f(x) = \frac{b}{a}\sqrt{a^2 - x^2}$



$\Rightarrow A_2 = \lim_{n \to \infty} \sum_{j=1}^{n} \frac{b}{a}\sqrt{a^2 - x_j^2}\left(\frac{a}{n}\right)$; donde $x_j = \frac{ja}{n}$. Es decir

$A_2 = \lim_{n \to \infty} \frac{b}{a} \sum_{j=1}^{n} \frac{a}{n}\sqrt{a^2 - \left(\frac{ja}{n}\right)^2}$. Pero

$\sum_{j=1}^{n} \frac{a}{n}\sqrt{a^2 - \left(\frac{ja}{n}\right)^2} = L(g, P_n)$, donde $g(x) = \sqrt{a^2 -}$

es la función que define $C \cap R$ $\Rightarrow$ $\lim_{n \to \infty} \frac{b}{a} \sum_{j=1}^{n} \frac{a}{n}\sqrt{a^2 - \left(\frac{ja}{n}\right)^2} =$
$\frac{b}{a} \lim_{n \to \infty} L(g, P_n) = \frac{b}{a}\left(\frac{\pi a^2}{4}\right)$ $\therefore$ $\text{Área}(\mathcal{E}) = 4\frac{b}{a}\left(\frac{\pi a^2}{4}\right) = \pi ab$.

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el radio del j-ésimo cilindro es

$$r_j = \sqrt{r^2 - \left(j\frac{r}{n}\right)^2}$$

$$V = \pi r_j^2 \frac{r}{n} = \pi\left(r^2 - j^2\left(\frac{r}{n}\right)^2\right)\frac{r}{n} = \pi\left(\frac{r^3}{n} - j^2\frac{r^3}{n^3}\right)$$

$$= \frac{\pi r^3}{n} - \pi j^2\left(\frac{r}{n}\right)^3$$

$$= \pi r^3\left(1 - \frac{j^2}{n^2}\right)\frac{1}{n}$$

$$V = \sum_{j=1}^{n} \pi r^3\left(1 - \frac{j^2}{n^2}\right)\frac{1}{n} = \pi r^3 \sum_{j=1}^{n} 1\cdot\frac{1}{n} - \sum_{j=1}^{n}\frac{j^2}{n^2}\cdot\frac{1}{n}$$

$$= \pi r^3\left[1 - \sum_{j=1}^{n}\frac{j^2}{n^3}\right.$$

$$= \pi r^3\left[1 - \frac{1}{n^3}\sum_{j=1}^{n} j^2\right]$$

$$= \pi r^3\left[1 - \frac{1}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]\right]$$

$$= \pi r^3\left[1 - \frac{1}{n^2}\frac{(n+1)(2n+1)}{6}\right]$$

$$= \pi r^3\left[1 - \frac{n}{n}\frac{n+1}{n}\frac{(2n+1)}{n}\frac{1}{6}\right]$$

$$\therefore\ V = 2\pi r^3\left[1 - 1\cdot\overset{\to 1}{\cancel{\frac{n+1}{n}}}\cdot\overset{\to 2}{\cancel{\frac{2n+1}{n}}}\cdot\frac{1}{6}\right]$$

$$\lim_{n\to\infty} V = 2\pi r^3\frac{4}{6} = \frac{4}{3}\pi r^3$$

$-\frac{1}{6} = \frac{5}{6}$

$\frac{5}{12}$