

SUCESIONES ∞

$$f(x) = x^2 + 7$$

$$(n^2 + 7)_{n \in \mathbb{N}}$$

DEF. SEA $a: \mathbb{N} \rightarrow \mathbb{R}$

UNA FUNCIÓN, SE DICE

QUE A ES UNA SUCESIÓN Y

SE DENOTA POR

$$(a_n)_{n \in \mathbb{N}} \text{ SIENDO } a_n$$

LA IMAGEN NATURAL DE LA

FUNCIÓN EVALUADA EN n ($a(n)$)

EJEMPLO

$$\left(\frac{2n-1}{n}\right)_{n \in \mathbb{N}}$$

SEA $n < m$ $n, m \in \mathbb{N}$

$$\frac{2m-1}{m} - \frac{2n-1}{n} = \frac{(2m-1)n - (2n-1)m}{nm}$$

$$= \frac{-nm}{nm} \quad \left. \begin{array}{l} \uparrow \\ \text{FUNCIÓN ES CRECIENTE} \end{array} \right\}$$

CBS. GENERALMENTE SE CONSIDERA

$$a \mathbb{N} = \{1, 2, \dots\}$$

$$2n \geq 2$$

$$\Rightarrow \frac{1}{n} \leq 1$$

$$\Rightarrow -\frac{1}{n} \geq -1$$

COTA INF

$$1 < n$$

$$0 < \frac{1}{n} \leq 1$$

$$0 > -\frac{1}{n} \geq -1$$

$$2 > 2 - \frac{1}{n} \geq 1$$

COTA SUP.

DEF. SEA $(a_n)_{n \in \mathbb{N}}$, $(b_n)_{n \in \mathbb{N}}$

DOS SUCESIONES, DEFINIDAS:

$$1 \quad (a_n + b_n)_{n \in \mathbb{N}} = a_n + b_n$$

$$2 \quad (\lambda a_n)_{n \in \mathbb{N}} = \lambda a_n$$

$$3 \quad (a_n b_n)_{n \in \mathbb{N}} = a_n b_n$$

$$4 \quad \left(\frac{a_n}{b_n}\right)_{n \in \mathbb{N}} = \frac{a_n}{b_n} \quad \text{si } b_n \neq 0 \quad \forall n \in \mathbb{N}$$

- TAREA MORAL - SOLUCIÓN DE LA PARADOJA DE AQUILES.
PARADOJAS DE XENON

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

$n \in \mathbb{N}$

n	1	2	10,000
$\frac{1}{n}$	1	$\frac{1}{2}$	$\frac{1}{10,000}$

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 7}{n^2 + 8n + 6} = \frac{2}{1}$$

$n \in \mathbb{N}$

$$\lim_{n \rightarrow \infty} \frac{2 + \frac{7}{n^2}}{1 + \frac{8}{n} + \frac{6}{n^2}} = \frac{2}{1} = 2$$

$$\lim_{n \rightarrow \infty} \frac{n!}{n^n} = \lim_{n \rightarrow \infty} \frac{n(n-1)(n-2)\dots(3)(2)(1)}{n(n/n)(n/n)\dots(n/n)} = 0$$

Arrows point from the denominator terms to 1, and from the final term to 0.

TABULAR $n=1$ $n=30$

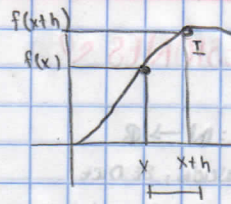
n	$\frac{n!}{n^n}$
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$$\lim_{n \rightarrow \infty} \frac{n^2 + 10}{\sqrt{n^4 + 7} - 1} = \frac{1}{1}$$

$$\lim_{n \rightarrow \infty} \frac{1 + \frac{10}{n^2}}{\sqrt{1 + \frac{7}{n^4}} - \frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{1 + \frac{10}{n^2}}{\sqrt{1 + \frac{7}{n^4}} - \frac{1}{n^2}} = 1$$

$$f(x) = x^2$$

$$f'(x) = 2x$$



$0 < h = \text{MUY PEQUEÑA}$

$$\frac{f(x+h) - f(x)}{x+h-x} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{(x+h)^2 - x^2}{h} = \frac{2xh + h^2}{h}$$

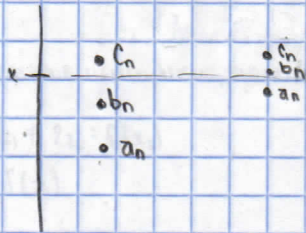
$$= 2x + h = 2x$$

TRES SUCEBIONES

$$a_n \leq b_n \leq c_n$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} c_n = k$$

$$\lim_{n \rightarrow \infty} b_n = k$$



DEF

$$\lim_{n \rightarrow \infty} a_n = L$$

si $\forall \epsilon > 0 \exists N \in \mathbb{N} \forall n \geq N$
 si $m \geq n \Rightarrow |a_m - a_n| < \epsilon$