Nombre:
Tema:

Día	Mes	Año	Folio
24	SEP	2015	1

DEF.

$\lim_{n\to\infty} a_n = L$

$\boxed{\begin{array}{l} \text{Si } \forall \varepsilon > 0 \ \exists_n \in \mathbb{N} \text{ tq} \\ \text{si } m > N \Rightarrow |a_m - L| < \varepsilon \end{array}}$

SOLUCIÓN DE $\lim_{n\to\infty} a_n = L$

$|x-3| < 7$

$\Rightarrow -7 < x-3 < 7$

$\Rightarrow -7+3 < x < 7+3$

$\lim_{n\to\infty} \frac{1}{n} = 0$

SEA $N = \lceil \frac{1}{\varepsilon} \rceil$

SI $m > N$

$\Rightarrow m > \lceil \frac{1}{\varepsilon} \rceil > \frac{1}{\varepsilon}$

$\Rightarrow \frac{1}{m} < \varepsilon$

$\Rightarrow |\frac{1}{m}| < \varepsilon$

$\therefore \lim_{n\to\infty} \frac{1}{n} = 0$

$\lim_{n\to\infty} \frac{n!}{n^n} = 0$

SEA $N = \lceil \frac{1}{\varepsilon} \rceil$

SI $m > N$

$\Rightarrow m > \lceil \frac{1}{\varepsilon} \rceil \geq \frac{1}{\varepsilon}$

$\Rightarrow \frac{1}{m} < \varepsilon$

$\Rightarrow \frac{m-k}{m} (\frac{1}{m}) < \frac{1}{m} < \varepsilon$

$\Rightarrow \frac{m-1}{m} (\frac{m-2}{m}) \cdots (\frac{1}{m}) < \varepsilon$

$\Rightarrow \frac{m}{m} (\frac{m-1}{m}) \cdots (\frac{1}{m}) < \varepsilon$

$\Rightarrow \frac{m}{m} (\frac{m-1}{m}) \cdots (\frac{1}{m}) < \varepsilon$

$\Rightarrow |\frac{m!}{m^n}| < \varepsilon$

OBS

$\forall_{k \in} (1 \ldots m-1)$

$m-k < m$

$\Rightarrow \frac{m-k}{m} < 1$

OBS

$\frac{1}{m} < \varepsilon$

$\frac{m-1}{m} (\frac{m-1}{m}) (\frac{1}{m}) < (\frac{m-1}{m}) \frac{1}{m} < \frac{1}{m} < \varepsilon$

$\lim_{n\to\infty} \frac{n+3}{n+7} = 1$

SEA $N = \lceil \frac{4}{\varepsilon} \rceil$

SI $m > N$

$\Rightarrow m > \frac{4}{\varepsilon}$

$\Rightarrow m+7 > m > \frac{4}{\varepsilon} \quad n+7 > n$

$\Rightarrow \frac{1}{m+7} < \frac{\varepsilon}{4}$

$\Rightarrow \frac{4}{m+7} < \varepsilon$

$\Rightarrow |\frac{m+3-m-7}{m+7}| < \varepsilon$

$\Rightarrow |\frac{m+3}{m+7} - 1| < \varepsilon$

BORRADOR:

$|\frac{n+3}{n+7} - 1| < \varepsilon$

$|\frac{n+3-n-7}{n+7}| < \varepsilon$

$|\frac{-4}{n+7}| < \varepsilon$

$|\frac{n+7}{4}| > \frac{1}{\varepsilon}$

$n+7 > \frac{4}{\varepsilon}$

$n > N$

TIP'S

$\boxed{\frac{m+3}{m+7} + \frac{-1(m+7)}{m+7}}$

Nombre:
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			2

$$\lim_{n\to\infty} \frac{an+b}{cn+d} = \frac{a}{c} \quad \text{si } c \neq 0$$

sea $N = \max\left[2\left\lceil \frac{|cb-ad|}{c^2\varepsilon}\right\rceil, 2\left\lceil \left|\frac{d}{c}\right| \right\rceil\right]$

Si $m > N$

$m > 2\left\lceil \frac{|cb-ad|}{c^2\varepsilon} \right\rceil > 2\left|\frac{cb-ad}{c^2\varepsilon}\right|$

$m > 2\left\lceil \left|\frac{d}{c}\right| \right\rceil > 2\left|\frac{d}{c}\right|$

$2m > 2\left[\left|\frac{cb-ad}{c^2\varepsilon}\right| + \left|\frac{d}{c}\right|\right]$

$\Rightarrow m > \left|\frac{cb-ad}{c^2\varepsilon}\right| + \left|\frac{d}{c}\right|$

$\Rightarrow m - \left|\frac{d}{c}\right| > \left|\frac{cb-ad}{c^2\varepsilon}\right|$

$\Rightarrow \left|m + \frac{d}{c}\right| > \left|\frac{cb-ad}{c^2\varepsilon}\right|$

$\Rightarrow c^2\left|m + \frac{d}{c}\right| > \frac{|cb-ad|}{\varepsilon}$

$\Rightarrow |c(cm+d)| > \frac{|cb-ad|}{\varepsilon}$

$\Rightarrow \frac{|cb-ad|}{|c(cm+d)|} < \varepsilon$

$\Rightarrow \left|\frac{cam+cb-cam-ad}{c(cm+d)}\right| < \varepsilon$

$\Rightarrow \left|\frac{c(an+b)-a(cm+d)}{c(cm+d)}\right| < \varepsilon$

BORRADOR

$\left|\frac{an+b}{cn+d} - \frac{a}{c}\right| < \varepsilon$

$\left|\frac{c(an+b) - a(cn+d)}{(cn+d)c}\right| < \varepsilon$

$\left|\frac{cb-ad}{(cn+d)c}\right| < \varepsilon$

$|(cn+d)c| > \frac{|cb-ad|}{\varepsilon}$

$\left|n + \frac{d}{c}\right| > \frac{|cb-ad|}{c^2\varepsilon}$

TEO.

SEAN $(a_n)_{n\in\mathbb{N}}$ y $(b_n)_{n\in\mathbb{N}}$ DOS SUCESIONES, C UNA CONSTANTE tq

① $\lim_{n\to\infty} a_n+b_n = A+B$

③ $\lim_{n\to\infty} a_n b_n = AB$

② $\lim_{n\to\infty} Ca_n = CA$

④ $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{A}{B}$

DEM

$\forall \varepsilon>0\ \exists N_A \in \mathbb{N}$ tq

$m > N_A \Rightarrow |a_n - A| < \varepsilon$

$\exists N_B \in \mathbb{N}$ tq

$m > N_B \Rightarrow |a_n - B| < \varepsilon$

SEA $\varepsilon = \frac{\varepsilon_0}{2} \Rightarrow \exists N_A, N_B \in \mathbb{N}$

SEA $N = \max(N_A, N_B)$ SI $m > N$

$\Rightarrow |a_n - A| < \frac{\varepsilon_0}{2}$ y $|b_n - B| < \frac{\varepsilon_0}{2}$

$\Rightarrow |a_n - A + b_n - B| \leq |a_n - A| + |b_n - B| < \varepsilon_0$

$\Rightarrow |(a_n+b_n) - (A+B)| < \varepsilon_0$

OBS

$\lim_{n\to\infty} a_n+b_n = A+B$

$\forall \varepsilon\ \exists N$ SI $m>N$

$|a_m+b_m - (A+B)| < \varepsilon$