Nombre:

Tema:

Día	Mes	Año	Folio
29	SEP	2018	1

SI $p>0$ $\lim\limits_{n\to\infty} \frac{1}{n^p} = 0$

DEM

$\forall \varepsilon>0 \quad N=\lceil (\frac{1}{\varepsilon})^{1/p} \rceil$

SI $n>N \geq (\frac{1}{\varepsilon})^{1/p}$

$\Rightarrow 0<\frac{1}{n}<\varepsilon^{1/p} \Rightarrow |(\frac{1}{n})^p|<\varepsilon$

$\Rightarrow (\frac{1}{n})^p < \varepsilon$

SI $p>0$ $\lim\limits_{n\to\infty} \sqrt[n]{p} = 1$

SI $p>0$ SEA $X_n = \sqrt[n]{p}-1$

POR LA DESIGUALDAD DE BERNOULLI

$1+nx_n \leq (1+x_n)^n = (1+\sqrt[n]{p}-1)^n$.

$\therefore 1+nX_n \leq p$

$\Rightarrow X_n \leq \frac{p-1}{n}$

SI $n\to\infty$ $X_n \to 0$

$\lim\limits_{n\to\infty} \sqrt[n]{p}-1=0$

$\Rightarrow \lim\limits_{n\to\infty} \sqrt[n]{p}=1$

$\lim\limits_{n\to\infty} \sqrt[n]{n}=1$

DEM $n\geq 1>0$.

$X_n = \sqrt[n]{n}-1$ POR EL TEOREMA DEL BINOMIO DE NEWTON

$(1+X_n)^n \geq 1+n(x_n)+\frac{n(n-1)}{2}(x_n)^2 \geq \frac{n(n-1)}{2}(x_n)^2 \Rightarrow X_n \leq \frac{p-1}{n}$

$\boxed{\text{OBS } n\geq 2}$

$n \geq \frac{n(n-1)}{2} \cdot (x_n)^2$

$\sqrt{\frac{2}{n-1}} \geq (x_n) \geq 0$

$\lim\limits_{n\to\infty} \geq \lim\limits_{n\to\infty} X_n \geq \lim\limits_{n\to\infty} 0$

$0 \geq \lim\limits_{n\to\infty} x_n \geq 0$

Nombre: | Día | Mes | Año | Folio
Tema: | | | | 2

$$\lim_{n\to\infty} \frac{n^k}{a^n} \quad (a>1 \;\; k>0) = 0$$

DEM

Si $k=1$ como $a>1 \Rightarrow \exists b>0$ tq

$$0 \le \frac{n}{a^n} = \frac{n}{(1+b)^n} \le \frac{n}{1+nb+\left(\frac{n(n-1)}{2}\right)b^2} \le \frac{n}{\frac{n(n-1)}{2}b^2}$$

$$= \frac{2}{(n-1)b^2}$$

BINOMIO DE NEWTON

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^{n-i} y^i$$

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

Si $p \ge 1$

$$\lim_{n\to\infty} \sqrt[n]{p} = 1$$

$0 < p < 1$

$\Rightarrow q = \frac{1}{p}$

$q > 1$

$$\Rightarrow \lim_{n\to\infty} \sqrt[n]{q} = 1 \neq 0$$

$$\lim_{n\to\infty} 1 = 1$$

$$\lim_{n\to\infty} \frac{1}{\sqrt[n]{q}} = 1$$

$$\frac{1}{\sqrt[n]{\frac{1}{p}}} = \frac{1}{\frac{1}{\sqrt[n]{p}}}$$

$$\left(\frac{1}{p}\right)^{\frac{1}{n}} = \frac{1^{\frac{1}{n}}}{p^{\frac{1}{n}}}$$

$$\lim_{n\to\infty} \sqrt[n]{p} = 1$$

$$p^{\frac{1}{k}} \ge 1$$

Si $r = \frac{k}{k+1} > 0$

$$\Rightarrow \left(p^{\frac{1}{k}}\right)^r \ge 1^r = 1$$

$$\Rightarrow \left(p^{\frac{1}{k+1}}\right) \ge 1$$