Clase del Sabado.

1 Integral de una función par:

Evaluar: $\int_{-2}^{2} (x^4 - 4x^2 + 6)\, dx$

Como $f(x) = x^4 - 4x^2 + 6$ es par en un intervalo simétrico

Tenemos que $f(-x) = f(x)$,

$[-2, 2]$, que:

$$\int_{-2}^{2} (x^4 - 4x^2 + 6)\, dx = 2\int_{0}^{2} (x^4 - 4x^2 + 6)\, dx =$$

$$= 2\left[\frac{x^5}{5} - \frac{4}{3}x^3 + 6x \right]_0^2 = \frac{232}{15}$$

2) $$\int \left(x^2 - \frac{1}{x^2}\right) dx = \int x^2\, dx - \int \frac{1}{x^2}\, dx = \frac{x^{2+1}}{2+1} - \int x^{-2}\, dx$$

$$= \frac{x^3}{3} - \frac{x^{-2+1}}{-2+1} + C = \frac{1}{3}x^3 + x^{-1} + C$$

$$= \frac{1}{3}x^3 + \frac{1}{x} + C$$

3) $$\int \frac{(2y+3)\, dy}{y} = \int \left(\frac{2y}{y} + \frac{3}{y}\right) dy = \int \left(2 + \frac{3}{y}\right) dy$$

$$= \int 2\, dy + \int \frac{3}{y}\, dy = 2\int dy + 3\int \frac{dy}{y}$$

$$= 2y + 3\ln|y| + C.$$

4) $$\int \frac{dx}{(x+1)^2}$$

Hacemos $u = x+1 \Rightarrow du = dx.$

$$\Rightarrow \int \frac{dx}{(x+1)^2} = \int \frac{du}{u^2} = \int u^{-2}\, du = \frac{u^{-1}}{-1} + C$$

$= -\frac{1}{u} + C$

Aplicamos de nuevo $u = x+1$ en la integral:

$= -\frac{1}{x+1} + C$.

5) $\int \frac{x\,dx}{x^2+2}$.

Sea $u = x^2+2 \Rightarrow du = 2x\,dx$

$\Rightarrow \frac{du}{2} = x\,dx$

$\Rightarrow \int \frac{x\,dx}{x^2+2} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$.

Sustituimos $u = x^2+2$

$\Rightarrow \int \frac{x\,dx}{x^2+2} = \frac{1}{2}\ln|x^2+2| + C$.

6) $\int \frac{2x+1}{x^2+x+1}\,dx$

Sea $u = x^2+x+1 \Rightarrow du = d(x^2+x+1) = 2x\,dx + dx + 0 = 2x\,dx + dx$

$= dx^2 + dx + d1 =$

$= (2x+1)\,dx \Rightarrow du = (2x+1)\,dx$.

$\Rightarrow \int \frac{2x+1}{x^2+x+1}\,dx = \int \frac{(2x+1)\,dx}{x^2+x+1}\,dx = \int \frac{du}{u} = \ln|u|$

Sea $u = x^2 + x + 1$

Sustituimos:

$$\int \frac{2x+1}{x^2+x+1}\,dx = \ln|x^2+x+1| + C.$$

Integración por sustitución:

7). $\int (\cos 3x + \operatorname{sen} 2x)\,dx = \int \cos 3x\,dx + \int \operatorname{sen} 2x\,dx.$

Hacemos:

$u = 3x$ $\quad w = 2x.$ para cada integral

$\therefore$ $du = 3dx$ $\quad dw = 2dx$

$\frac{du}{3} = dx$ $\quad \frac{dw}{2}\ dx.$

$$\Rightarrow \int \cos 3x\,dx + \int \operatorname{sen} 2x\,dx = \int \cos u \frac{du}{3} + \int \operatorname{sen} w \frac{dw}{2} = \frac{1}{3}\int \cos u\,d$$

$$+ \frac{1}{2}\int \operatorname{sen} u\,du$$

$$= \frac{1}{3}\operatorname{sen} u - \frac{1}{2}\cos w + C.$$

Pero $u = 3x$ $\quad w = 2x.$

$$\therefore \int (\cos 3x + \operatorname{sen} 2x)\,dx = \frac{1}{3}\operatorname{sen} 3x - \frac{1}{2}\cos 2x + C.$$

8) $\int \sec^2 \frac{1}{3} \theta \, d\theta$.

Hacemos $u = \frac{1}{3}\theta$.

$\Rightarrow du = \frac{1}{3} d\theta \quad \Rightarrow \quad 3du = d\theta$

Sustituimos:

$\int \sec^2 \frac{1}{3} \theta \, d\theta = \int \sec^2 u \, (3du) = 3\int \sec^2 u \, du$

$= 3 \operatorname{tg} u + C$

Pero $u = \frac{1}{3}\theta$.

$\Rightarrow \int \sec^2 \frac{1}{3} \theta \, d\theta = 3 \operatorname{tg} \frac{1}{3} \theta + C$.

9) $\int \frac{\operatorname{sen} x \, dx}{\cos^4 x} = \int \frac{\operatorname{sen} x}{\cos x} \cdot \frac{1}{\cos^3 x} \, dx = \int \operatorname{tg} x \sec^3 x \, dx$

$= \int \sec^2 x \operatorname{tg} x \sec x \, dx$

Hacemos $u = \sec x$

$du = \operatorname{tg} x \sec x \, dx$.

Sustituimos:

$= \int u^2 \, du = \frac{u^3}{3} + C \qquad \frac{\sec^3 x}{3} + C$.

$\Rightarrow \int \frac{\sec x \, dx}{\cos^4 x} =$

10) $\int \frac{(\text{Cos}x - \text{Sen}x)^2}{\text{Sen}x} dx.$

Desarrollamos el binomio al cuadrado:

$$\int \frac{\text{Cos}^2 x - 2\text{Cos}x\text{Sen}x + \text{Sen}^2 x}{\text{Sen}\,x} dx.$$

Dividiendo entre Senx.

$$\int \left(\frac{\text{Cos}^2 x}{\text{Sen}\,x} - \frac{2\,\text{Cos}x\text{Sen}x}{\text{Sen}\,x} + \frac{\text{Sen}^2 x}{\text{Sen}x} \right) dx.$$

$$= \int \left(\frac{\text{Cos}^2 x}{\text{Sen}x} - 2(\text{Cos}x + \text{Sen}x) \right) dx.$$

Distribuyendo la integral a cada sumando.

$$= \int \frac{\text{Cos}^2 x}{\text{Sen}x} dx - \int 2\,\text{Cos}\,x\, dx + \int \text{sen}x\, dx.$$

$$= \int \frac{(1 - \text{Sen}^2 x)}{\text{Sen}x} dx - 2\int \text{Cos}x\, dx + \int \text{Sen}x dx.$$

$$= \frac{1}{\text{Sen}\,x} dx - \int \frac{\text{Sen}^2 x}{\text{Sen}x} dx - 2\,(\text{Sen}x) - \text{Cos}\,x$$

$$= \int \text{csc}\,x\, dx - \int \text{Sen}x\, dx - 2\text{Sen}x - \text{Cos}x$$

$$= \ln\,(\text{csc}x - \text{ctg}\,x) + \text{Cos}x - 2\,\text{Sen}x - \text{Cos}\,x + C$$

$$\Rightarrow \int \frac{(\text{Cos}x - \text{Sen}x)^2}{\text{Sen}x} dx = \ln\,(\text{csc}x - \text{ctg}\,x) - 2\,\text{Sen}\,x + C$$

11) $\int \frac{\csc^2 x\,dx}{1+2\,ctg x}$

Sea $u = 1 + 2\,ctg x$

$du = 2\,d\,ctg x = 2(-\csc^2 x)\,dx.$

$du = -2\csc^2 x\,dx$

$\therefore\; -\frac{du}{2} = \csc^2 x\,dx.$

$\Rightarrow \int \frac{\csc^2 x\;dx}{1+2\,ctg x} = -\frac{1}{2}\int \frac{du}{u} = -\frac{1}{2}\ln|u| + c$

$= -\frac{1}{2}\ln|(1+2ctgx)| + c$

12) $\int \frac{dx}{x\sqrt{2x^2-3}} = \int \frac{\sqrt{2}\;dx}{\sqrt{2}x\sqrt{2x^2-3}} = \sqrt{2}\int \frac{dx}{\sqrt{2}x\sqrt{2x}}$

Sea $u^2 = 2x^2$ $\quad a^2 = 3$

$u = \sqrt{2}x$ $\quad a = \sqrt{3}$

$du = \sqrt{2}\,dx.$

Sustituimos:

$\int \frac{\sqrt{2}\;dx}{\sqrt{2}x\sqrt{2x^2-3}} = \int \frac{du}{u\sqrt{u^2-a^2}}$

$= \frac{1}{a}\,arcSec\,\frac{u}{a} + C = \frac{1}{\sqrt{3}}\,arcSec\,\frac{\sqrt{2x}}{\sqrt{3}} + c$

13) $\int \frac{e^x\,dx}{1+e^{2x}}$

Sea $u^2 = e^{2x}$ $\qquad a^2 = 1$

$u = (e^{2x})^{1/2} = e^x$

$du = e^x\,dx$

Sustituimos:

$$\int \frac{e^x\,dx}{1+e^{2x}} = \int \frac{du}{a^2+u^2} = \frac{1}{a}\,\text{arc tg}\,\frac{u}{a} + C$$

$= \text{arctg}\,(e^x) + C$

Más ejercicios:

14) $\int x\,\text{Sen}(x^2+1)\,dx = -\frac{1}{2}\,\text{Cos}(x^2+1) + C$

15) $\int \frac{2\,dx}{4x^2+3} = \frac{1}{\sqrt{3}}\,\text{arc tg.}\,\frac{2x}{\sqrt{3}} + C$

16) $\int \frac{dx}{x\sqrt{2x^2-3}} = \frac{1}{\sqrt{3}}\,\text{arc. Sen}\,\frac{\sqrt{2}x}{\sqrt{3}} + C$

17) $\int \frac{dy}{4y^2+3} = \frac{1}{2}\cdot\frac{1}{\sqrt{3}}\,\text{arc.tg.}\,\frac{2y}{\sqrt{3}} + C$

18)

19) $\int \frac{x+1}{\sqrt{x^2+4}}\,dx = \sqrt{x^2+4} + \ln(x+\sqrt{x^2+4}) + C$

20)

21) $\int \frac{dx}{x^2+6x+13} = \frac{1}{2}\,\text{arc. tg}\,\frac{x+3}{2} + C$

22) $\int \frac{dx}{(x-1)\sqrt{x^2-x-3}} = \frac{1}{2}\,\text{arc. Sec.}\,\frac{x-1}{2} + C$

23)

24) $\int \frac{e^x\,dx}{e^{2x}+2e^x+3} = \frac{1}{\sqrt{2}}\,\text{arc. tg}\,\frac{e^x+1}{\sqrt{2}} + C$

25)

26) $\int \text{Sen}^3 3\theta\,\text{Cos}^3 3\theta\,d\theta = \frac{\text{Sen}^4 3\theta}{12} - \frac{\text{Sen}^6 3\theta}{18} + C$

27) $\int \cos x \operatorname{Sen} 2x \, dx = -\frac{1}{6} \cos 3x - \frac{1}{2} \cos x + C$

28) $\int \cos^2 (4x) \, dx = \frac{1}{2} x + \frac{1}{16} \operatorname{Sen} 8x + C.$

29) $\int \cos^2 x \operatorname{Sen}^4 x \, dx = \frac{x}{16} - \frac{\operatorname{Sen} 4x}{64} - \frac{\operatorname{Sen}^3 2x}{48} + C$

30) $\int \sqrt{1 + \cos\theta} \, d\theta = 2\sqrt{2} \operatorname{Sen} \frac{\theta}{2} + C.$

31) $\int x^3 \sqrt{x^2 + a^2} \, dx = \frac{1}{15} (3x^2 - 2a^2)(x^2 + a^2)^{3/2} + C.$

32) $\int \frac{2z \, dz}{\sqrt[3]{z^2 + 1}} = \frac{3}{2} (z^2 + 1)^{2/3} + C$

33) $\int x^2 \operatorname{Sen}(x^3) \, dx = -\frac{1}{3} \cos(x^3) + C.$

34) $\int_{-1}^{1} |x| \, dx = 1$

35) $\int (x^3 + x)^5 (3x^2 + 1) \, dx = \frac{(x^3 + x)^6}{6} + C.$

37) $\int_{-1}^{1} 3x^2 \sqrt{x^3 + 1} \, dx = \frac{4\sqrt{2}}{3}$

38) $\int_{\pi/4}^{\pi/2} \cot\theta \csc^2\theta \, d\theta = \frac{1}{2}.$

39) $\int_0^3 2x \sqrt{9 - x^2} \, dx = 18.$

40) $\int_1^4 \pi [\sqrt{x} - 1]^2 \, dx = \frac{7\pi}{6}$