01/04/2014

Ejercicios

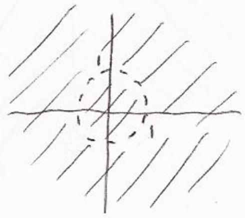
1) Determinar dominio y rango de las siguientes funciones:

a) $f(x,y) = \dfrac{1}{x^2+y^2-1}$

b) $f(x,y) = \dfrac{1}{\sqrt{x^2+y^2-1}}$

c) $f(x,y,z) = z\sqrt{x^2+y^2-25}$

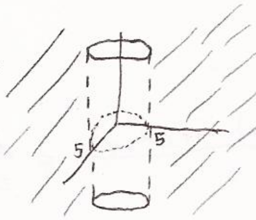
Sol.

a) $\text{Dom} f = \{(x,y)\in\mathbb{R}^2 \mid x^2+y^2 \neq 1\} \longrightarrow$ 

$\text{Ran} f = (-\infty,-1] \cup (0,\infty)$

b) $\text{Dom} f = \{(x,y)\in\mathbb{R}^2 \mid x^2+y^2 > 1\} \longrightarrow$ 

$\text{Ran} f = (0,\infty)$

c) $\text{Dom} f = \{(x,y,z)\in\mathbb{R}^3 \mid x^2+y^2 \geq 25\} \longrightarrow$ 

$\text{Ran} f = (-\infty,\infty)$

2) Dibujar las curvas de nivel de las siguientes funciones:

a) $f(x,y) = x^2+y^2$

b) $f(x,y) = \sqrt{1-x^2-y^2}$

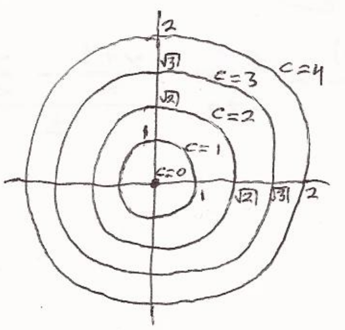
c) $f(x,y) = x^2-y^2$

Sol.

a) $\text{Dom} f = \mathbb{R}^2$

$\text{Ran} f = [0,\infty)$

$x^2+y^2 = c$



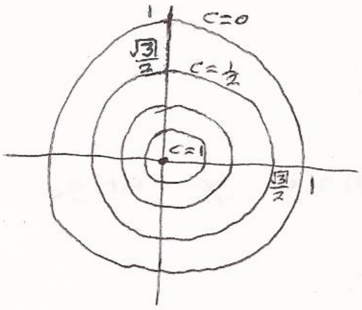
círculos concéntricos en el origen con radios crecientes conforme $c \to \infty$

b) $\mathrm{Dom} f = \{(x,y)\in \mathbb{R}^2 \mid x^2+y^2 \leq 1\}$

$\mathrm{Ran} f = [0,1]$

$c = \sqrt{1-x^2-y^2}$

$\Rightarrow \; x^2+y^2 = 1-c^2$

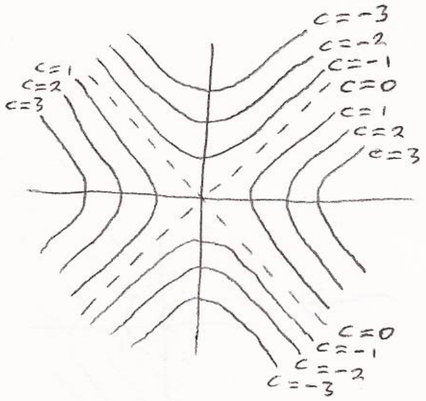


círculos concéntricos en el origen con radios decrecientes conforme $c \to 1$.

c) $\mathrm{Dom} f = \mathbb{R}^2$

$\mathrm{Ran} f = (-\infty,\infty)$

$x^2-y^2 = c$



Hipérbolas con vértices en el eje y $(c<0)$ y en el eje x $(c>0)$, con asíntotas en $y=x$ y $y=-x$ $(c=0)$

3) Calcula las derivadas parciales de las siguientes funciones usando la definición:

a) $f(x,y) = xy$

b) $f(x,y) = \dfrac{y}{x}$

Sol.

a) $\dfrac{\partial f}{\partial x} = \lim\limits_{t\to 0} \dfrac{f(x+t,y)-f(x,y)}{t} = \lim\limits_{t\to 0} \dfrac{(x+t)y - xy}{t} = \lim\limits_{t\to 0} \dfrac{\cancel{xy}+y\cancel{t}-\cancel{xy}}{\cancel{t}} = y$

$\dfrac{\partial f}{\partial y} = \lim\limits_{t\to 0} \dfrac{f(x,y+t)-f(x,y)}{t} = \lim\limits_{t\to 0} \dfrac{x(y+t)-xy}{t} = \lim\limits_{t\to 0} \dfrac{\cancel{xy}+x\cancel{t}-\cancel{xy}}{\cancel{t}} = x$

b) $\dfrac{\partial f}{\partial x} = \lim\limits_{t\to 0} \dfrac{f(x+t,y)-f(x,y)}{t} = \lim\limits_{t\to 0} \dfrac{\frac{y}{x+t}-\frac{y}{x}}{t} = \lim\limits_{t\to 0} \dfrac{\frac{\cancel{xy}-\cancel{xy}-yt}{x(x+t)}}{t}$

$= \lim\limits_{t\to 0} \dfrac{-y\cancel{t}}{x(x+t)\cancel{t}} = -\dfrac{y}{x^2}$

$\dfrac{\partial f}{\partial y} = \lim\limits_{t\to 0} \dfrac{f(x,y+t)-f(x,y)}{t} = \lim\limits_{t\to 0} \dfrac{\frac{y+t}{x}-\frac{y}{x}}{t} = \lim\limits_{t\to 0} \dfrac{\cancel{\frac{y}{x}}+\frac{t}{x}-\cancel{\frac{y}{x}}}{\cancel{t}} = \dfrac{1}{x}$

01/04/2014

4) Calcula las derivadas parciales de las siguientes funciones:

a) $f(x,y,z) = \operatorname{sen}\left(\frac{\ln(1+x^2)}{1+e^{z^2+y}}\right)$

b) $f(x,y,z) = z^{2x} \arctan yx^2$

c) $f(x,y) = \ln(\tan \frac{x}{y})$

d) $f(x,y) = \operatorname{senh}\left(\frac{1}{\sqrt{x^2+y^2}}\right)$

e) $f(x,y) = \int_{xy}^{xy} g(t)\,dt$

<u>Sol.</u>

a) $\frac{\partial f}{\partial x} = \cos\left(\frac{\ln(1+x^2)}{1+e^{z^2+y}}\right)\left(\frac{2x}{1+e^{z^2+y}}\right)\left(\frac{1}{1+x^2}\right)$

$\frac{\partial f}{\partial y} = \cos\left(\frac{\ln(1+x^2)}{1+e^{z^2+y}}\right)\left(\frac{\ln(1+x^2)}{(1+e^{z^2+y})^2}\right)\left(-e^{z^2+y}\right)$

$\frac{\partial f}{\partial z} = \cos\left(\frac{\ln(1+x^2)}{1+e^{z^2+y}}\right)\left(\frac{\ln(1+x^2)}{(1+e^{z^2+y})^2}\right)\left(-e^{z^2+y}\right)(2z)$

b) $\frac{\partial f}{\partial x} = (2\ln z)\, z^{2x} \arctan yx^2 + z^{2x}\left(\frac{2xy}{1+y^2x^4}\right)$

$\frac{\partial f}{\partial y} = \frac{x^2 z^{2x}}{1+y^2x^4}$

$\frac{\partial f}{\partial z} = 2x\, z^{2x-1} \arctan yx^2$

c) $\frac{\partial f}{\partial x} = \left(\cot \frac{x}{y}\right)\left(\frac{1}{y}\right)$

$\frac{\partial f}{\partial y} = \left(\cot \frac{x}{y}\right)\left(-\frac{x}{y^2}\right)$

d) $\frac{\partial f}{\partial x} = \cosh\left(\frac{1}{\sqrt{x^2+y^2}}\right)\left(\frac{-x}{(x^2+y^2)^{3/2}}\right)$

$\frac{\partial f}{\partial y} = \cosh\left(\frac{1}{\sqrt{x^2+y^2}}\right)\left(\frac{-y}{(x^2+y^2)^{3/2}}\right)$

e) $\frac{\partial f}{\partial x} = g(xy)\frac{\partial (xy)}{\partial x} - g(x^y)\frac{\partial (x^y)}{\partial x} = y\,g(xy) - y x^{y-1} g(x^y)$

$\frac{\partial f}{\partial y} = g(xy)\frac{\partial (xy)}{\partial y} - g(x^y)\frac{\partial (x^y)}{\partial y} = x\,g(xy) - \ln x\,(x^y)\,g(x^y)$