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Sea $f(t)=(1, e^t \operatorname{sen} t, e^t \cos t)$

Calcule $T(t)$, $N(t)$, $B(t)$, plano osculador, plano normal, plano rectificable y curvatura en $t_0=0$

$f'(t)=(0, e^t(\operatorname{sen}t+\cos t), e^t(\cos t-\operatorname{sen}t))$

$f''(t)=(0, e^t(\cancel{\operatorname{sen}t}+\cos t+\cos t-\cancel{\operatorname{sen}t}), e^t(\cancel{\cos t}-\operatorname{sen}t-\operatorname{sen}t-\cancel{\cos t}))$

$=(0, 2e^t\cos t, -2e^t\operatorname{sen}t)$

$\|f'(t)\| = \sqrt{e^{2t}(\operatorname{sen}^2t+\cancel{2\operatorname{sen}t\cos t}+\cos^2t+\cos^2t-\cancel{2\operatorname{sen}t\cos t}+\operatorname{sen}^2t)} = \sqrt{2}e^t$

$\Rightarrow T(t)=\dfrac{f'(t)}{\|f'(t)\|}=\dfrac{1}{\sqrt{2}}(0, \operatorname{sen}t+\cos t, \cos t-\operatorname{sen}t)$ y $T(t_0)=\dfrac{1}{\sqrt{2}}(0,1,1)$

Ahora, $T'(t)=\dfrac{1}{\sqrt{2}}(0, \cos t-\operatorname{sen}t, -\operatorname{sen}t-\cos t)$

$\|T'(t)\| = \sqrt{\frac{1}{2}(\cos^2t-\cancel{2\cos t\operatorname{sen}t}+\operatorname{sen}^2t+\operatorname{sen}^2t+\cancel{2\operatorname{sen}t\cos t}+\cos^2t)}=1$

$\Rightarrow N(t)=\dfrac{T'(t)}{\|T'(t)\|}=\dfrac{1}{\sqrt{2}}(0, \cos t-\operatorname{sen}t, -\operatorname{sen}t-\cos t)$ y $N(t_0)=\dfrac{1}{\sqrt{2}}(0,1,-1)$

$B(t_0)=T(t_0)\times N(t_0)=\dfrac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & 1\\ 0 & 1 & -1\end{vmatrix}=\dfrac{1}{2}(-2,0,0)=(-1,0,0)$

$P_0=f(t_0)=(1,0,1)$

$\Rightarrow$ Plano osculador $\Pi_0=(P-P_0)\cdot B(t_0)=(x-1, y, z-1)\cdot(-1,0,0)=1-x=0$

$\therefore$ $x=1$

Plano normal $\Pi_N=(P-P_0)\cdot T(t_0)=(x-1,y,z-1)\cdot\left(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)=\frac{y}{\sqrt{2}}+\frac{z-1}{\sqrt{2}}=0$

$\therefore$ $y+z-1=0$

Plano rectificable $\Pi_R=(P-P_0)\cdot N(t_0)=(x-1,y,z-1)\cdot\left(0,\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)=\frac{y}{\sqrt{2}}+\frac{1-z}{\sqrt{2}}=0$

$\therefore$ $y-z+1=0$

Por último:

$K(t)=\dfrac{\|f'(t)\times f''(t)\|}{\|f'(t)\|^3}$ $\Rightarrow$

$$f'(t)\times f''(t)=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & e^{t}(\operatorname{Sen}t+\cos t) & e^{t}(\cos t-\operatorname{Sen}t) \\ 0 & 2e^{t}\cos t & -2e^{t}\operatorname{Sen}t \end{vmatrix}$$

$$=(-2e^{2t}(\operatorname{Sen}^2 t+\operatorname{Sen}t\cos t)-2e^{2t}(\cos^2 t-\operatorname{Sen}t\cos t),\ 0,\ 0)$$

$$=(-2e^{2t},0,0)$$

$\Rightarrow \|f'(t)\times f''(t)\| = 2e^{2t}$ ya que $e^{2t}>0\ \forall t$

$\therefore\ K(t)=\dfrac{2e^{2t}}{(\sqrt{2}e^{t})^3}=\dfrac{e^{-t}}{\sqrt{2}}$ y $K(t_0)=\dfrac{1}{\sqrt{2}}$