3) $\forall v_0 \alpha \to \exists v_0 \alpha$

sea $\mathfrak{A} \in V_p$ y $s \in {}^{w}A$

PDJ $\mathfrak{A} \models \forall v_0 \alpha \to \exists v_0 \alpha\, [s]$

syss$_{Tarski}$ $\mathfrak{A} \not\models \forall v_0 \alpha [s]$ $\mathfrak{A} \models \exists v_0 \alpha [s]$

[recordando $A \to B \equiv \neg A \vee B$]

syss (si $\mathfrak{A} \models \forall v_0 \alpha [s]$ en $\mathfrak{A} \models \exists v_0 \alpha [s]$)

$\therefore$ basta demostrar que

PDJ $\mathfrak{A} \models \forall v_0 \alpha [s]$ implica que $\mathfrak{A} \models \exists v_0 \alpha [s]$

dem) suponsumos $\mathfrak{A} \models \forall v_0 \alpha [s]$, espo pasa syss$_{Tarski}$

para toda $a \in A$ $(\mathfrak{A} \models \alpha [s(^{v_0}/_{a})])$

<u>como $A \neq \emptyset$</u> ent. hay $a_0 \in A$ t.q

$\mathfrak{A} \models \alpha [s(^{v_0}/_{a_0})]$ syss$_{Tarski}$ <u>$\mathfrak{A} \models \exists v_0 \alpha [s]$</u>

$\therefore$ $\mathfrak{A} \models \exists v_0 \alpha [s]$

$\therefore$ si $\mathfrak{A} \models \forall v_0 \alpha [s]$ ent $\mathfrak{A} \models \exists v_0 \alpha [s]$

$\therefore$ $\mathfrak{A} \models \forall v_0 \alpha \to \exists v_0 \alpha\, [s]$

como $\mathfrak{A}$ y s son arbitrarias

$\forall v_0 \alpha \to \exists v_0 \alpha$ es U.V.

4) $\forall v_0(\alpha \dagger \beta) \to ((\forall v_0 \alpha) \dagger (\forall v_0 \beta))$

sean $\mathfrak{A} \in V_\rho$ $\quad s \in {}^{\omega}A$

$\mathfrak{A} \models \forall v_0(\alpha \dagger \beta) \to ((\forall v_0 \alpha) \dagger (\forall v_0 \beta))[s]$ $\quad syss_{Tarski}$

$\mathfrak{A} \not\models \forall v_0(\alpha \dagger \beta)[s]$ ó $\mathfrak{A} \models (\forall v_0 \alpha) \dagger (\forall v_0 \beta)[s]$ $\quad$ syss

[recordando $\quad A \to B \equiv \neg A \vee B$]

si $\mathfrak{A} \models \forall v_0(\alpha \dagger \beta)[s]$ $\quad$ ent $\mathfrak{A} \models (\forall v_0 \alpha) \dagger (\forall v_0 \beta)[s]$

supongamos $\quad \mathfrak{A} \models \forall v_0(\alpha \dagger \beta)[s]$

notemos que $\quad \mathfrak{A} \models (\forall v_0 \alpha) \dagger (\forall v_0 \beta)[s]$ $\quad syss_{Tarski}$

$\mathfrak{A} \models \forall v_0 \alpha[s]$ y $\mathfrak{A} \models \forall v_0 \beta[s]$ $\quad syss_{Tarski}$

p.t. $a \in A$ $\quad \mathfrak{A} \models \alpha[s(^{v_0}/_a)]$ y $\mathfrak{A} \models \forall v_0 \beta[s]$ $\quad syss_{Tarski}$

p.t. $a, b \in A$ $\quad \mathfrak{A} \models \alpha[s(^{v_0}/_a)]$ y $\mathfrak{A} \models \beta[s(^{v_0}/_b)]$

PD) p.t. $a, b \in A$ $\quad \mathfrak{A} \models \alpha[s(^{v_0}/_a)]$ y $\mathfrak{A} \models \beta[s(^{v_0}/_b)]$

dem) sean $a, b \in A$, como $\mathfrak{A} \models \forall v_0(\alpha \dagger \beta)[s]$ ent (Tarski)

p.t. $c \in A$ $(\mathfrak{A} \models \alpha \dagger \beta[s(^{v_0}/_c)])$ $\quad syss_{Tarski}$

p.t. $c \in A$ $(\mathfrak{A} \models \alpha[s(^{v_0}/_c)]$ y $\mathfrak{A} \models \beta[s(^{v_0}/_c)])$

en particular, para $c = a$ $\quad \mathfrak{A} \models \alpha[s(^{v_0}/_a)]$

y para $c = b$ $\quad \mathfrak{A} \models \beta[s(^{v_0}/_b)]$

$\therefore$ $\mathfrak{A} \models \alpha[s(^{v_0}/_a)]$ y $\mathfrak{A} \models \beta[s(^{v_0}/_b)]$, que es lo que queríamos probar

$\therefore$ $\mathfrak{A} \models \forall v_0(\alpha \dagger \beta) \to ((\forall v_0 \alpha) \dagger (\forall v_0 \beta))[s]$

como $\mathfrak{A}$ y s son arbitrarias

$\forall v_0(\alpha \dagger \beta) \to ((\forall v_0 \alpha) \dagger (\forall v_0 \beta))$ es U.V.